Eur. Phys. J. C 49, 869–873 (2007)
DOI 10.1140/epjc/s10052-006-0155-9

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Birth of the universe from the landscape of string theory

A. Kobakhidze[a], L. Mersini-Houghton

Department of Physics and Astronomy, UNC-Chapel Hill, NC 27599-3255, USA

Received: 24 August 2006 /
Published online: 29 November 2006 –

Abstract. We show that a unique, most probable and stable solution for the wavefunction of the universe, with a very small cosmological constant $\Lambda_1 \simeq \left(\frac{\pi}{l_\text{p}N}\right)^2$, can be predicted from the supersymmetric minisuperspace with N vacua of the landscape of string theory without referring to the anthropic principle. Due to the nearest neighbor tunneling in moduli space lattice, the N-fold degeneracy of the vacua is lifted and a discrete spectrum of bound state levels over the whole minisuperspace emerges. Supersymmetry is spontaneously broken by these bound states, with discrete non-zero energy levels $\Lambda_s \simeq \left(\frac{s\pi}{l_\text{p}N}\right)^2$, $s = 1, 2, \ldots$

PACS. 98.80.Qc; 11.25.Wx

Recent progress in string theory has revealed a large and rich structure of vacuum solutions in moduli space [1–11], known as the landscape [3]. The large number of vacua results from the fact that in a typical compactification of M-theory from eleven dimensions to $(3+1)$ dimensions there are hundreds of ways of 'wrapping' compact dimensions with flux and hundreds of 4-form fields and fluxes. Counting of string theory vacua has been the subject of much recent works on landscape theory [8–11]. The (Poincaré) supersymmetric (SUSY) vacua are degenerate with zero vacuum energy, while the non-SUSY part of the landscape is expected to have vacua with different but finite values of the energy density λ in the range 0–M_p^4. In general one expects to have disconnected sectors of such vacua as well.

The potential $V(\phi)$ of the moduli field ϕ (which describes the collective contributions from all moduli ϕ_i), is typically described as having many valleys (the vacuum solutions) separated by barriers with height of order M_p^4. Though the detailed structure of the landscape is not yet fully understood, the large number of vacuum solutions will quite likely persist. As a result the following question has been the central theme in the landscape investigation: *In which vacuum, from this multitude of choices, does our universe reside*? There seems to be no physical selection criterion to answer this challenge, a fact that has led many people to seek support from anthropic arguments instead [3].

Our approach here is entirely different from the way this challenge has been explored so far. We do not ask 'in which vacuum do we live?'. Instead we are interested in finding the answer to the following question: *Which stable solution of the wavefunction of the universe is the most probable solution over the whole superspace of the landscape?* As we show below, the wavefunction of the universe actually spreads over the whole landscape rather than being localized around a certain particular vacuum. We model the landscape by considering the superspace of moduli ϕ as a finite lattice of N sites with tunneling between the sites taken into account in the nearest neighbor approximation. We show that there is a discrete range of solutions that form 'energy' bands, and the most probable one is the minimum energy bound state which is lifted from zero. Thus, our treatment predicts a universe having a small cosmological constant, $\Lambda \simeq \left(\frac{\pi}{l_\text{p}N}\right)^2$ (provided N is large), as the most probable one, without referring to the anthropic arguments.

1 The minisuperspace approach

In the superspace of all vacuum solutions for moduli ϕ with homogeneous 3-geometries, we consider only one sector, namely, the minisuperspace of SUSY vacua, that preserve R-parity symmetry, described by the potential $V(\phi)$ with potential wells that sit at zero, and by the metric of spatially flat and homogeneous 3-geometries

$$\mathrm{d}s^2 = [-\mathcal{N}\,\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\mathbf{x}^2]\,, \tag{1}$$

$\mathcal{N}$ is a lapse function that can be set to $\mathcal{N} = 1$. We also make the assumption of equal a priori probabilities for each SUSY vacuum to occur. This means that we consider the potential $V(\phi)$ for the modulus field to have a periodic 'lattice' type distribution of equidistant potential wells and

[a] e-mail: kobakhid@physics.unc.edu

barriers. The energy of the SUSY vacua will sit at $\lambda_i = 0$, and for the barrier heights and spacing b between them we can take the typical values of $\mathcal{O}(M_{\rm p}^4)$ and Planck length $b \simeq l_{\rm p}$ respectively. Let the number of (vacuum) sites in this SUSY minisuperspace 'periodic lattice' be some large but finite number N. The SUSY minisuperspace is defined by the configuration space of the two variables ϕ and a. The wavefunction of the universe propagating through the SUSY minisuperspace, $\Psi(a,\phi)$, is a functional over the configurations ϕ and a, as we switch on gravity. All our calculations and results below can easily be extended to closed and open universes.

2 Formalism

We set out to solve the Wheeler–De Witt equation in the minisuperspace. The combined action of the a, ϕ background is

$$S = S_{\rm g} + S_\phi = \int {\rm d}^4x\sqrt{-g}\left[\frac{R}{\kappa^2} + \frac{\dot\phi^2}{2} - \frac{1}{2}V(\phi)\right]. \quad (2)$$

We set below the normalization factor $\kappa^2 = 12$ and the Planck constant $\hbar$ equal to one, unless otherwise noted. The Lagrangian for gravity is

$$L_{\rm g} = \frac{1}{2}\mathcal{N}\left[-\frac{a\dot a^2}{\mathcal{N}^2} - a^3\Lambda\right]. \quad (3)$$

The canonical momenta are defined by $p_a = \frac{\partial L_{\rm g}}{\partial \dot a} = -\frac{a\dot a}{\mathcal{N}}$. The corresponding Hamiltonian becomes

$$\mathcal{H}_{\rm g} = (p_a\dot a - L_{\rm g})/\mathcal{N} = -\frac{1}{2a}\left[p_a^2 - a^4\Lambda\right]. \quad (4)$$

The homogeneous moduli field is described by the Lagrangian

$$L_\phi = \frac{a^3\mathcal{N}}{2}\left(\frac{\dot\phi^2}{\mathcal{N}} - V(\phi)\right), \quad (5)$$

and its canonical momenta are given by $p_\phi = \frac{\partial L_\phi}{\partial \dot\phi} = a^3\dot\phi$. The potential for the moduli ϕ in the SUSY minisuperspace is some periodic function with zero energy for all N potential wells and lattice spacing b as explained above, which satisfies

$$V(\phi) = V(\phi + b), \quad (6)$$

with barrier heights $\simeq O(M_{\rm p}^4)$. The Hamiltonian for the field, with $\mathcal{N}$ set to one, is

$$\mathcal{H}_\phi = \frac{a^3}{2}\left(\dot\phi^2 + V(\phi)\right). \quad (7)$$

Let us define $\ln(a) = \alpha$, $\frac{\dot a}{a} = \dot\alpha$. The full Hamiltonian in the (α,ϕ) minisuperspace on the SUSY sector of the superspace with $\Lambda = \lambda_i = 0$ becomes

$$\mathcal{H} = \frac{1}{2{\rm e}^{3\alpha}}\left[-p_\alpha^2 + p_\phi^2 + {\rm e}^{6\alpha}V(\phi)\right]. \quad (8)$$

The system is quantized by promoting the conjugate momenta in (8) to the operators $\hat p_\alpha = -{\rm i}\frac{\partial}{\partial\alpha}$ and $\hat p_\phi = -{\rm i}\frac{\partial}{\partial\phi}$.[1] We have

$$\hat{\mathcal{H}} = \frac{1}{2{\rm e}^{3\alpha}}\left[\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2} + {\rm e}^{6\alpha}V(\phi)\right]. \quad (9)$$

The Wheeler–DeWitt equation is the quantum Hamiltonian constraint, obtained by varying the action (2) with respect to $\mathcal{N}$,

$$\hat{\mathcal{H}}\Psi(a,\phi) = 0, \quad (10)$$

with the Hamiltonian operator $\mathcal{H}$ given by (9) acting on the wavefunction of the universe $\Psi(\alpha,\phi)$.

The field equations of motion are obtained by varying the action, S in (2), with respect to α and ϕ, respectively, and read

$$\ddot\phi + 3H\dot\phi + \frac{1}{2}\frac{\partial V}{\partial\phi} = 0, \quad (11)$$

$$\ddot\alpha + \frac{3}{2}[\dot\alpha^2 + \dot\phi^2 - V(\phi)] = 0. \quad (12)$$

It is easy to check for consistency that in fact the α equation of motion is nothing more than the Friedmann equation for the expansion in the presence of the energy density of the field ϕ, $\epsilon = \frac{\dot\phi^2}{2} + \frac{1}{2}V(\phi)$.

3 Boundary conditions and solution

The SUSY minisuperspace periodic lattice contains a large but finite number of 'lattice sites' (vacua), N, in the potential $V(\phi)$, (6). We will assume that there is no interaction with the 'hidden sectors' of the superspace, and thus the wavefunction does not leak out to other sectors. The solution to the Wheeler–DeWitt equation (10) would produce an N-fold degenerate ground state for $\Psi(\alpha,\phi)$. Let us now allow for tunneling between sites, in the nearest neighbhor approximation, with a tunneling rate δ. Finite large periodic lattices have been extensively studied in condensed matter physics [21]. Tunneling between neighbor sites lifts the degeneracy of the vacuum solution as it breaks the lattice translation symmetry. In order to establish the analogy with condensed matter systems, let us for the moment take $\alpha = \text{constant}$ in (9). For our SUSY 'lattice' the ground state vacuum energy is $\Lambda = 0$. The boundary conditions in this large but finite lattice quantize the wavenumber k in terms of a discrete quantum number s. There are two possible boundary conditions we can choose for the N lattice sites numbered 0 to $N-1$: *The fixed end-point* boundary that requires the wavefunction does not propagate outside the minisuperspace, i.e. the function should vanish at these end-points; or *the cyclic boundary condition* relevant

[1] There is the operator ordering ambiguity which can be ignored in the semiclassical approximation we are interested in here. For more details and subtleties of the minisuperspace approach to the second quantization of gravity see, e.g. [12–18].

for large N which requires that the sth site should satisfy $\Psi_{N+s} = \Psi_s$. For large N the two are equivalent by the symmetry of $k \to -k$, $|k| \le \frac{\pi}{L}$ and $L = bN$. There are $N-1$ normal modes in this configuration. Boundary conditions thus require that

$$k_s = \frac{\pi s}{bN}\,, \quad s = 1, 2, \ldots, N\,. \tag{13}$$

Due to the mixing between nearest neighbors from tunneling, the Hamiltonian has non-diagonal terms. Diagonalizing the Hamiltonian yields the energy eigenvalues of (15), thereby splitting the levels and removing the N-fold degeneracy of the ground state. The eigenfunctions obtained after the diagonalization of the Hamiltonian are the normal modes of the system given by $\Psi_{k(s)}(\phi) \simeq \sin(k_s\phi)$ or $\cos(k_s\phi)$. Physically these eigenfunctions are a superposition of left and right moving Bloch plane waves which, due to constructive interference in their phases, satisfy the Bragg reflection condition and form standing waves in the minisuperspace lattice of size $L = bN$. These bound states are *extended* states on the landscape as they can spread over many vacua due to tunneling. The expressions for the standing waves consistent with the boundary condition are

$$\Psi_s \simeq \frac{\sin(k_s\phi)}{\sqrt{k_s}}\,, \tag{14}$$

where the quantum number s (not to be confused with the lattice site numbering), takes values in the range $s = 1, \ldots, N$. The eigenvalues of the Hamiltonian form bands of energy with discrete energy levels, ϵ_s. A rough estimate for the tunneling rate can be given by $\delta \simeq \left(\frac{\pi}{b}\right)^2$, known as the mass gap of periodic lattices. The energy of each level with wavenumber k_s is

$$\epsilon_s = 2\delta - 2\delta\cos(k_s b)\,. \tag{15}$$

At this point we re-establish the dependence on α and find solutions for the wavefunction of the universe $\Psi(\alpha,\phi)$ to the full Hamiltonian given by (9). Let us take the following ansatz for the wavefunction of the universe Ψ in (10):

$$\Psi(\alpha, x) = \Sigma_s F_{k_s}(\alpha)\psi_{k_s}(x)\,. \tag{16}$$

We have rescaled the variable and the parameters as follows: ϕ to $x = \mathrm{e}^{3\alpha}\phi$, $\tilde{b} = b\mathrm{e}^{3\alpha}$, $\tilde{k}_s = k_s\mathrm{e}^{-3\alpha}$, $\tilde{\delta} = \delta\mathrm{e}^{6\alpha}$ so that (10) becomes separable in α, x. After rescaling, $\psi_{k_s}(x)$ in (16) satisfies the α independent equation

$$\left[-\frac{\partial^2}{\partial x^2} + V(x)\right]\psi_{k_s}(x) = \epsilon_{k_s}\psi_k(x)\,. \tag{17}$$

The energy eigenvalues $\epsilon_s \simeq \frac{\hbar^2 k_s^2}{2}$ (as in (15)) and the solutions for the eigenfunctions ψ_{k_s} are given by (13) and (14).

The lowest energy standing wave is the one for $s = 1$, $\tilde{k}_1 = \frac{\pi}{bN}$, $\epsilon_1 = \left(\frac{\pi}{bN}\right)^2$. By plugging (17) back into (10) we obtain that the $F_{k_s}(\alpha)$ of (16) satisfy the following equation:

$$\left[\frac{\partial^2}{\partial\alpha^2} + \mathrm{e}^{6\alpha}\epsilon_s\right]F_{k_s}(\alpha) = 0\,. \tag{18}$$

The solution to this equation is a zero order Bessel function,

$$\begin{aligned} F_{k_s}(\alpha) &= J_0\left(\frac{1}{3}\sqrt{\epsilon_s}\mathrm{e}^{6\alpha}\right) \\ &= \frac{1}{2}\left(H_0^{(1)}\left(\frac{1}{3}\sqrt{\epsilon_s}\mathrm{e}^{6\alpha}\right) + H_0^{(2)}\left(\frac{1}{3}\sqrt{\epsilon_s}\mathrm{e}^{6\alpha}\right)\right)\,, \end{aligned} \tag{19}$$

namely the Hankel function $H_0^{(2)}$ with the chosen boundary conditions. In the large $a = \mathrm{e}^{\alpha}$ limit it reads

$$F_s(\alpha) \approx \frac{1}{(|\tilde{\epsilon}_s|)^{1/4}}\mathrm{e}^{\pm\mathrm{i}\sqrt{|\tilde{\epsilon}_s|}a^3}\,, \tag{20}$$

where $\tilde{\epsilon}_s = \mathrm{e}^{6\alpha}\epsilon_s = \left(\frac{s\pi}{bN}\right)^2$ with $s = 1, 2, \ldots, N$.

The solution to the equation of motion for α, (12), yields $\alpha = \pm|\epsilon_s|^{1/2}t = \pm(H_s t)$. The growing mode soon dominates over the decaying one, thus we take only the outgoing mode $\alpha = +H_s t$ as our boundary condition at time plus infinity (see [14] for details). Therefore, each standing wave mode labelled by the quantum number s in the expression (16) for the wavefunction $\Psi(\alpha,\phi)$ describes a de Sitter universe with its own constant non-zero cosmological constant $\tilde{\epsilon}_s \simeq \left(\frac{\pi s}{bN}\right)^2$, time and expansion rate $\alpha = +H_s t$. There are $N-1$ discrete normal modes that form the discrete energy band of bound states, all lifted from zero by the respective level energy ϵ_s. Clearly this mass gap of levels spontaneously breaks the SUSY of the background landscape. Decoherence between levels is resolved since the energy levels are discrete and separated by a finite amount of energy. The lowest lying energy state, corresponding to $s = 1$, has a non-zero energy of $\tilde{\epsilon}_1 = \left(\frac{\pi}{bN}\right)^2$.

There is an ongoing debate in quantum cosmology [14] on the measure of probability in the wavefunction of the universe, both definitions being plagued with some pathologies. The probability for (9), viewed as a Klein–Gordon equation, is given by $\mathcal{P} = \mathrm{i}(\Psi\partial_a\Psi^* - \Psi^*\partial_a\Psi)$. The same Hamiltonian, when treated with the quantum mechanic formalism has a probability given by $\mathcal{P} = |\Psi|^2$. Due to the oscillatory solution for the modes of $\Psi(\alpha, x)$, in our case both expressions for the probability give, up to an overall normalization constant, $\left(\left(\frac{b}{\pi}\right)^2\right.$ for lattices$\left.\right)$

$$\mathcal{P} \approx \frac{1}{|\tilde{\epsilon}_s|}\,. \tag{21}$$

This shows that $\mathcal{P}$ is peaked around Ψ_1 with energy $\tilde{\epsilon}_1 \approx \left(\frac{\pi}{bN}\right)^2$. Although the SUSY landscape potential has $\lambda = 0$, our calculation shows that the most probable solution, (21), is peaked around the first bound state in the discrete band of energy levels, i.e. the lowest lying energy level $s = 1$. As shown this lowest lying level has a non-zero energy constant energy $\tilde{\epsilon}_1 \approx \left(\frac{\pi}{bN}\right)^2$. The lifting of the degeneracy of the N vacua and thus the spontaneous breaking of SUSY by the bound state Ψ_1 due to tunneling gives birth to a universe with a small cosmological constant $\Lambda = H_1^2 = \epsilon_1$. N is expected to be large enough. Thus having $\epsilon_1 \approx \Lambda$ in the favored range of $\lambda \approx 10^{-120}M_{\mathrm{p}}^4$ can be easily arranged.

Assuming R-symmetry is not crucial or restrictive to our result. Removing the R-symmetry consideration from the SUSY sector of the landscape would extend this sector by allowing the AdS type vacua with $\lambda \leq 0$. Hence including the AdS vacua in the superspace does not change our results. The SUSY AdS solutions result in a term $\lambda < 0$ in (4) which may render $F_s(\alpha)$, (20), to be a decaying solution for all ϵ_s for which $\epsilon_s + \lambda < 0$. These solutions have vanishing probability and do not give birth to a universe; therefore they are physically irrelevant. This shows that the most probable solution still is the first bound state lifted *above zero*. We have just shown that Ψ_1 is a unique, stable and most probable solution with non-zero energy propagating in the SUSY minisuperspace of the landscape. It can therefore be a candidate for the wavefunction of the universe from the landscape.

4 Concluding remarks

We have shown here how a unique, most probable and stable solution for the wavefunction of the universe can be predicted from the landscape without having to appeal to anthropic arguments. The solution found here spontaneously breaks SUSY, since it contains a small and non-zero $\Lambda = \left(\frac{\pi}{l_{\mathrm{p}} N}\right)^2$. Due to tunneling, the wavefunction can spread throughout the landscape rather than be localized around one vacuum site. As we showed, Ψ_s are extended states. This result is a radical departure from the point of view taken in the literature, where the question “Which vacua do we live in?” implies a highly localized solution for Ψ_s. Constructive interference between right and left moving plane waves for the large but *finite* landscape gives rise to $N-1$ normal mode bound states that occupy the SUSY minisuperspace of the landscape in a discrete band of energy levels that have a non-zero mass gap from the degenerate vacuum $\lambda = 0$. The non-zero mass gap for the wavefunction of the universe is responsible for the spontaneous breaking of the SUSY of the landscape. Each of the bound states has its own energy and can give rise to a de Sitter universe with a different expansion rate according to the energy level. The probability for the bound state levels is inversely proportional to their energy; thus the lowest lying energy state becomes the most probable one. It is interesting to note that we get a different estimate for the ground state energy dependence on N; namely, we obtained $\Lambda \simeq N^{-2}$ instead of the current estimate appearing in the literature i.e. $\Lambda \simeq N^{-1}$. It should be noted that the $N \to \infty$ limit recovers SUSY on the 3-geometry as it makes the mass gap Λ for the energy level disappear.

Thus, according to our picture, the universe nucleates directly from the quantum gravity era with a small cosmological constant $\Lambda = \left(\frac{\pi}{bN}\right)^2$. We have assumed here that the subsequent evolution of the universe is the conventional one. We would need to extend our minisuperspace to include more degrees of freedom such as the matter content for the 3-geometries, in order to account for a more realistic universe and to be able to address issues such as inflation, reheating, mass hierarchy and others. These important issues are not trivial within the proposed scenario and deserve further investigation. We hope to report our result on the extension of the landscape minisuperspace to 3-geometries that contain the standard model.

Finally, we would like to stress that our scenario is very different from that of [19] (see also [20] for a different approach to the cosmological constant problem within the theories with degenerate vacua). There, one assumed the existence of a coherent superposition of landscape vacua, similar to the *theta*-vacuum in Yang–Mills theory. However, *theta*-vacua in field theories appear only if there are stable classical field configurations with finite action (instantons) which provide for the inter-vacua tunnelings. In contrast to [19], we are working in the context of quantum cosmology on minisuperspace and our equation (17) describes the quantum mechanical problem of a particle propagating in a periodic potential. The solution to this problem is by the well-known Bloch waves. Thus in our scenario the role of the *theta*-vacuum is played by the Bloch wave type solution (see (16)) for the wavefunction of the universe. Each mode in (16) describes a universe with a certain cosmological constant (and supposedly other physical ‘observables’) to which we can assign quantum mechanical probabilities as is discussed above. This kind of dynamic selection criterion, based on the probability calculation, may be a more physical alternative to the statistical [8–11] or anthropic selection criteria for the landscape vacua. Obviously there is a transition from quantum to classical cosmology which involves decoherence. We assume that once our universe (the one with the largest quantum mechanical probability) is born during the “quantum era”, will decohere, and the subsequent evolution will be determined by the standard classical cosmology.

Acknowledgements. We are very grateful to Laurie McNeil and Dmitri Khevchenko for many beneficial discussions about condensed matter analogs. We would also like to thank Andreas Albrecht for stimulating and helpful comments.

References

1. R. Bousso, J. Polchinski, JHEP **0006**, 006 (2000) [hep-th/0004134]
2. S. Kachru, R. Kallosh, A. Linde, S.P. Trivedi, Phys. Rev. D **68**, 046 005 (2003) [hep-th/0301240]
3. L. Susskind, hep-th/0302219
4. L. Susskind, hep-th/0405189
5. B. Freivogel, L. Susskind, hep-th/0408133
6. T. Banks, M. Dine, E. Gorbatov, JHEP **0408**, 058 (2004) [hep-th/0309170]
7. M. Dine, E. Gorbatov, S. Thomas, hep-th/0407043
8. M.R. Douglas, JHEP **0305**, 046 (2003) [hep-th/0303194]
9. F. Denef, M.R. Douglas, B. Florea, JHEP **0406**, 034 (2004) [hep-th/0404257]
10. M.R. Douglas, hep-th/0405279
11. M.R. Douglas, hep-th/0409207
12. J.B. Hartle, S.W. Hawking, Phys. Rev. D **28**, 2960 (1983)
13. T. Vachaspati, A. Vilenkin, Phys. Rev. D **37**, 898 (1988)

14. A. Vilenkin, Phys. Rev. D **50**, 2581 (1994) [gr-qc/9403010] and references therein
15. J.J. Halliwell, Phys. Rev. D **64**, 044 008 (2001) [gr-qc/0008046]
16. J.J. Halliwell, Phys. Rev. D **63**, 085 013 (2001) [quant-ph/0011103]
17. D.L. Wiltshire, gr-qc/0101003
18. M.J.W. Hall, K. Kumar, M. Reginatto, J. Phys. A **36**, 9779 (2003) [hep-th/0307259]
19. G.L. Kane, M.J. Perry, A.N. Zytkow, hep-ph/0408169
20. J. Yokoyama, Phys. Rev. Lett. **88**, 151 302 (2002) [hep-th/0110137]
21. C. Kittel, Introduction to Solid State Physics (John Wiley and Sons Inc., 1968)